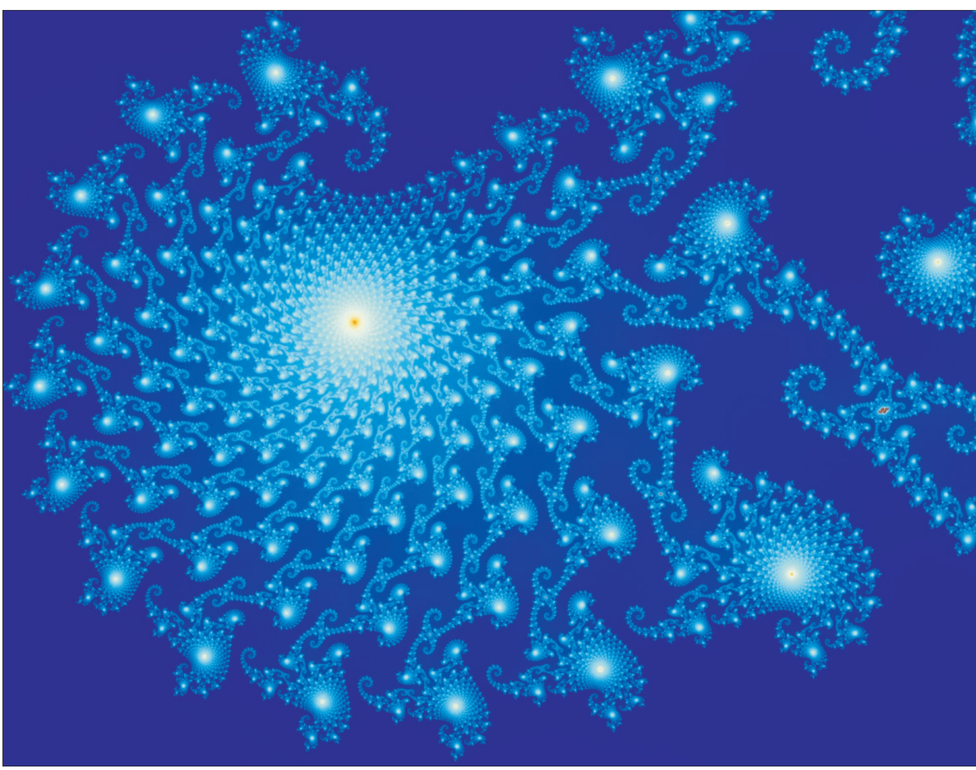
Number Theory and Algebraic Equations

Odile Marie-Thérèse Pons



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Preface

The factorization of every integer as a product of its prime divisors is the basis of the arithmetics and the search of rules to compute them for large numbers is the origin of the elementary number theory, it is related to the factorization of polynomials. Fermat's theorems and his comments on the Diophantine equations are well-known mathematical texts of the XVII-th century, they are connections between arithmetics and geometry through equations for the intersections of elliptic curves or geometric solids. They are generally studied separately in algebra and analysis. Most proofs of the number theory rely on field properties and Galois theory, their applications to the geometry send them back to their origin.

Many early publications have stated properties of numbers without proofs and the attempts to solve them remained unsuccessful for a long time. Fermat's last theorem concerning the non existence of non zero integer solutions of the equations

$$x^n + y^n = z^n$$

was unsolved untill recently, partial proofs suppose the knowledge of the existence of roots of homogeneous polynomials of degree n but the premices have not been assertened. The extension of theorems with primes to composite numbers is often difficult and intricated, many questions are not closed.

The conjectures of the separation of integers, polynomials or curves as sums or products of elements belonging to specific families are still open challenges though they may be computationally validated on large scales in the related fields. The main results of this domain have been proved during the XVIII-th century by Euler, Lagrange, Legendre, Gauss, Dirichlet and others with the development of the modern algebra. After the publication of tables of prime number during the previous century, they published tables of prime polynomials.

Riemann's hypothesis about the complex zeros of Euler's function ζ is another conjecture, it is connected to the evaluation of the number $\pi(x)$ of primes until x and to expansions of the functions $\pi(x)$ and $\zeta(x)$.

The book is an undergraduate and graduate course for students in mathematics, the concepts are illustrated by many formal and numerical examples. It covers the classical number theory and new generalizations are introduced. My proofs are often simpler than the classical ones, they include an elementary proof of Fermat's last theorem and extensions. Chapter 1 introduces the reader to the representation and the clasification of the numbers. Chapter 2 focuses on Fermat's first theorem, its extensions and applications to the quadratic residues and the representation of integers as sums of squares. Chapter 3 develops solutions of Diophantine equations and algebraic equations, I prove Fermat's last theorem, Catalan's conjecture and study equations that extend Fermat's theorem such as Dirichlet's equations for $x^5 + y^5$. Chapter 4 deals with the distributions of the prime integers and the factorization of integer polynomials, Chapter 2-4 are much indebt to Legendre writings. The functions Gamma and zeta and other series are related to the cardinal of the primes, Chapter 5 recalls their properties with detailed proofs of their expansions. Chapter 6 focuses on the algebraic number theory in fields of decomposition of irreductible polynomials and Galois's groups. Chapter 7 applies the field theory to polynomials and functional problems, throughout examples illustrate the theory.

Odile M.-T. Pons
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Contents